

Validity of single-channel model for a spin-orbit coupled atomic Fermi gas near Feshbach resonances

Jing-Xin Cui^{1,2}, Xia-Ji Liu², Gui Lu Long^{1,3}, and Hui Hu^{2*}

¹*Department of Physics, Tsinghua University, Beijing 100084, China*

²*ARC Centre of Excellence for Quantum-Atom Optics,*

Centre for Atom Optics and Ultrafast Spectroscopy,

Swinburne University of Technology, Melbourne 3122, Australia

³*Tsinghua National Laboratory for Information Science and Technology, Tsinghua University, Beijing 100084, China*

(Dated: January 23, 2013)

We theoretically investigate a Rashba spin-orbit coupled Fermi gas near Feshbach resonances, by using mean-field theory and a two-channel model that takes into account explicitly Feshbach molecules in the close channel. In the absence of spin-orbit coupling, when the channel coupling g between the closed and open channels is strong, it is widely accepted that the two-channel model is equivalent to a single-channel model that excludes Feshbach molecules. This is the so-called broad resonance limit, which is well-satisfied by ultracold atomic Fermi gases of ^6Li atoms and ^{40}K atoms in current experiments. Here, with Rashba spin-orbit coupling we find that the condition for equivalence becomes much more stringent. As a result, the single-channel model may already be insufficient to describe properly an atomic Fermi gas of ^{40}K atoms at a moderate spin-orbit coupling. We determine a characteristic channel coupling strength g_c as a function of the spin-orbit coupling strength, above which the single-channel and two-channel models are approximately equivalent. We also find that for narrow resonance with small channel coupling, the pairing gap and molecular fraction is strongly suppressed by SO coupling. Our results can be readily tested in ^{40}K atoms by using optical molecular spectroscopy.

I. INTRODUCTION

As a realization of non-abelian gauge fields in neutral cold atoms [1–5], spin-orbit (SO) coupled atomic gases have attracted a lot of attentions in recent years. The SO coupled bosonic gas of ^{87}Rb atoms was first achieved by Spielman's group at National Institute of Standards and Technology (NIST) in early 2011 [1]. The SO coupled atomic Fermi gas has also been realized most recently at Shanxi University [4] and at Massachusetts Institute of Technology (MIT) [5] with ^{40}K and ^6Li atoms, respectively. These novel atomic gases have many interesting properties inherent to spin-orbit coupling, and have potential applications in future quantum technology. A well-known example is the emulation of the long-sought topological superfluids and Majorana fermions [6–8], which lie at the heart of topological quantum information and computation [9, 10].

Most of previous theoretical studies on SO coupled Fermi gases are based on a single-channel model [6, 11–31]. In this model, the interaction between atoms is described by a single parameter, i.e., the s -wave scattering length a_s . The scattering length can experimentally be tuned by using Feshbach resonances. As a result, in the absence of SO coupling, the Fermi gas can cross from a Bose-Einstein condensate (BEC) over to a Bardeen-Cooper-Schrieffer (BCS) superfluid [32], when the scattering length changes from positive to negative values. With SO coupling, the picture of BEC-BCS crossover

may be qualitatively altered. For example, for a particular Rashba-type SO coupling, a new two-body bound state - referred to as rashbon - is formed [11–14]. By increasing the SO coupling strength, the system may change from a BCS superfluid to a BEC of rashbons, even on the BCS side with a negative scattering $a_s < 0$ [12–14]. The pairing gap of this system will be significantly enhanced due to the increased density of state at the Fermi surface [14]. An anisotropic superfluid due to the Rashba SO coupling has also been predicted [13].

A more realistic and complete description of ultracold atomic Fermi gases near Feshbach resonances, however, should be the two-channel model, which includes both atoms in the open channel and Feshbach molecules in the closed channel [33]. In this model, in addition to the background s -wave scattering length a_{bg} between atoms, two other parameters are used in order to fully describe the interaction. These are the detuning energy of Feshbach molecules ν and the channel coupling strength g between molecules and Fermi atoms. Therefore, the interaction of the system consists of two parts. The non-resonant part is the contact interaction between atoms with the strength determined by the background scattering length, while the resonant interaction is induced by the coupling between molecules and atoms. Near Feshbach resonances without SO coupling, it is known that the single-channel and two-channel models are essentially equivalent when the channel coupling strength g is large enough [34, 35]. This is the so-called broad resonance condition, satisfied by the Fermi gases of ^{40}K and ^6Li atoms, which are so far the two main systems used in the cold-atom laboratory.

In this paper, we aim to examine the equivalence of

*Electronic address: hhu@swin.edu.au

the single-channel and two-channel models for a Rashba SO coupled Fermi gas near Feshbach resonances. This is by no means obvious, as fermionic pairing is notably affected by SO coupling at the BEC-BCS crossover. We use mean-field theory and focus on the most interesting resonant limit. Our results show that in the presence of SO coupling, the broad resonance condition is much more difficult to achieve. As a result, for an ultracold atomic Fermi gas of ^{40}K atoms, which is known to be well described by the single-channel model without SO coupling, we may have to use a two-channel model already at a moderate SO coupling strength.

Our paper is organized as follows. In the next section (Sec. II), we introduce the model Hamiltonian. In Sec. III, we diagonalize the Hamiltonian by using mean-field theory to obtain the grand thermodynamic potential and solve the resulting coupled mean-field equations. In Sec. IV, we discuss the equivalence between the single-channel and two-channel models. In Sec. V, we show how to test experimentally the difference between the two models, by using optical molecular spectroscopy [36]. Finally, we summarize in Sec. VI.

II. MODEL HAMILTONIAN

We consider a three-dimensional (3D) resonantly-interacting atomic Fermi gas with Rashba-type SO coupling, described by the two-channel model Hamiltonian,

$$\mathcal{H} = \mathcal{H}_{SO} + \mathcal{H}_m + \mathcal{H}_I, \quad (1)$$

where \mathcal{H}_{SO} and \mathcal{H}_m stand for the non-interacting Hamiltonian of SO coupled atoms in the open channel and of Feshbach molecules in the closed channel, respectively. The interaction Hamiltonian $\mathcal{H}_I = \mathcal{H}_{am} + \mathcal{H}_{aa}$ includes both the atom-molecule coupling between the two channels (\mathcal{H}_{am}) and the background interaction between open-channel atoms (\mathcal{H}_{aa}).

For atoms, we take the following single-particle Rashba SO Hamiltonian,

$$\mathcal{H}_{SO} = \frac{\hbar^2 \mathbf{k}^2}{2m} + \frac{\hbar^2}{2m} \lambda \mathbf{k}_\perp \cdot \boldsymbol{\sigma}_\perp, \quad (2)$$

where $\mathbf{k}_\perp \equiv (k_x, k_y)$ and $\boldsymbol{\sigma}_\perp \equiv (\sigma_x, \sigma_y)$ are respectively the in-plane momentum and in-plane Pauli matrix, and λ is the Rashba SO coupling strength. Note that, the standard representation of the Rashba SO coupling is given by $\lambda(k_y \sigma_x - k_x \sigma_y)$ [13]. Here, for convenience we have performed a spin-rotation to rewrite the Rashba term into a slightly different but fully equivalent form $\lambda(k_x \sigma_x + k_y \sigma_y)$ [14]. In the second quantized form,

$$\mathcal{H}_{SO} = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} a_{\mathbf{k}\sigma}^\dagger a_{\mathbf{k}\sigma} + \frac{\hbar^2 \lambda k_\perp}{2m} \left(e^{-i\varphi_{\mathbf{k}}} a_{\mathbf{k}\uparrow}^\dagger a_{\mathbf{k}\downarrow} + \text{H.c.} \right), \quad (3)$$

where $a_{\mathbf{k}\sigma}^\dagger$ is the creation operator for atoms with momentum \mathbf{k} in the spin state σ , $\epsilon_{\mathbf{k}} \equiv \hbar^2 \mathbf{k}^2 / (2m)$ and $\varphi_{\mathbf{k}} \equiv \arg(k_x + ik_y)$. To diagonalize this single-particle Hamiltonian, we introduce the field operators in the helicity basis labeled by “ \pm ”,

$$\begin{pmatrix} h_{\mathbf{k}+} \\ h_{\mathbf{k}-} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{-i\varphi_{\mathbf{k}}} \\ e^{i\varphi_{\mathbf{k}}} & -1 \end{pmatrix} \begin{pmatrix} a_{\mathbf{k}\uparrow} \\ a_{\mathbf{k}\downarrow} \end{pmatrix}, \quad (4)$$

with which the single-particle Rashba Hamiltonian becomes diagonal,

$$\mathcal{H}_{SO} = \sum_{\mathbf{k}} \left(\epsilon_{\mathbf{k}+} h_{\mathbf{k}+}^\dagger h_{\mathbf{k}+} + \epsilon_{\mathbf{k}-} h_{\mathbf{k}-}^\dagger h_{\mathbf{k}-} \right). \quad (5)$$

Note that, in the helicity basis, the single-particle dispersion relation now breaks into two branches: $\epsilon_{\mathbf{k}+} = \epsilon_{\mathbf{k}} + \hbar^2 \lambda k_\perp / (2m)$ for the upper branch and $\epsilon_{\mathbf{k}-} = \epsilon_{\mathbf{k}} - \hbar^2 \lambda k_\perp / (2m)$ for the lower branch. To describe Feshbach molecules, we use annihilation operators $b_{\mathbf{q}}$. The energy of molecules is denoted as 2ν , which after renormalization [33] is related to the detuning energy from threshold of Feshbach resonance B_0 , i.e., $2\nu_0 = \Delta\mu(B - B_0)$, where $\Delta\mu \equiv 2\mu_a - \mu_m$ is the magnetic moment difference between the atomic ($2\mu_a$) and bound molecular state (μ_m) [33]. The Hamiltonian of Feshbach molecules may be written as,

$$\mathcal{H}_m = 2\nu \sum_{\mathbf{q}} b_{\mathbf{q}}^\dagger b_{\mathbf{q}}. \quad (6)$$

Finally, the interaction Hamiltonian is given by $\mathcal{H}_I = \mathcal{H}_{aa} + \mathcal{H}_{am}$, where

$$\mathcal{H}_{aa} = U_{bg} \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}} a_{\mathbf{q}/2+\mathbf{k}\uparrow}^\dagger a_{\mathbf{q}/2-\mathbf{k}\downarrow}^\dagger a_{\mathbf{q}/2-\mathbf{k}'\downarrow} a_{\mathbf{q}/2+\mathbf{k}'\uparrow} \quad (7)$$

is the non-resonant interaction between atoms, with strength given by the background s -wave scattering length after renormalization [33], $U_{bg} = 4\pi\hbar^2 a_{bg}/m$, and

$$\mathcal{H}_{am} = g \sum_{\mathbf{k}\mathbf{q}} [b_{\mathbf{q}}^\dagger a_{\mathbf{q}/2+\mathbf{k}\uparrow} a_{\mathbf{q}/2-\mathbf{k}\downarrow} + \text{H.c.}] \quad (8)$$

is the resonant interaction between atoms and molecules, with strength parameterized by g . After renormalization, the magnitude of the channel coupling strength g is related to the width of the Feshbach resonance W , i.e., $g \equiv \sqrt{\Delta\mu W U_{bg}}$. We note that, in the two-channel model one may define an effective s -wave length [35],

$$a_s = a_{bg} \left(1 - \frac{W}{B - B_0} \right) = a_{bg} - \frac{g^2}{2\nu_0} \frac{m}{4\pi\hbar^2}. \quad (9)$$

III. MEAN FIELD THEORY

We use the standard mean-field theory to solve the two-channel model Eq. (1), by assuming that all the molecules and Cooper pairs condense into the zero-momentum state. Thus, we set $\mathbf{q} = 0$ in the interaction Hamiltonian \mathcal{H}_I . Here, we have excluded the possibility of an inhomogeneous superfluid phase (i.e., $\mathbf{q} \neq 0$), which may exist in the presence of an in-plane Zeeman field [37]. This is consistent with the two-body calculation [13, 27, 38] that the ground state of two particles in our Hamiltonian always has zero center-of-mass momentum. Following the procedure in Ref. [33], we introduce the following field parameters:

$$\phi_m = \langle b_0 \rangle, \quad (10)$$

$$p = \sum_{\mathbf{k}} \langle a_{\mathbf{k}\uparrow} a_{-\mathbf{k}\downarrow} \rangle, \quad (11)$$

$$f = \sum_{\mathbf{k}} \langle a_{\mathbf{k}\uparrow}^\dagger a_{\mathbf{k}\uparrow} \rangle = \sum_{\mathbf{k}} \langle a_{\mathbf{k}\downarrow}^\dagger a_{\mathbf{k}\downarrow} \rangle, \quad (12)$$

where ϕ_m is the molecular field in the closed channel, p is the pairing field, and f is half of the number of fermionic atoms in the open channel. The interaction Hamiltonian \mathcal{H}_{aa} can therefore be written as

$$\frac{\mathcal{H}_{aa}}{U_{bg}} \simeq \sum_{\mathbf{k}\sigma} f a_{\mathbf{k}\sigma}^\dagger a_{\mathbf{k}\sigma} - \sum_{\mathbf{k}} \left(p a_{\mathbf{k}\uparrow}^\dagger a_{-\mathbf{k}\downarrow}^\dagger + \text{H.c.} \right) - |p|^2 - f^2. \quad (13)$$

Similarly, we approximate \mathcal{H}_{am} as

$$\mathcal{H}_{am} \simeq -g \sum_{\mathbf{k}} \left(\phi_m a_{\mathbf{k}\uparrow}^\dagger a_{-\mathbf{k}\downarrow}^\dagger + \text{H.c.} \right). \quad (14)$$

Thus, within mean-field the total Hamiltonian is given by,

$$\begin{aligned} \mathcal{H} = & -U_{bg} (|p|^2 + f^2) + \sum_{\mathbf{k}\tau} \epsilon_{\mathbf{k}\tau} h_{\mathbf{k}\tau}^\dagger h_{\mathbf{k}\tau} + \sum_{\mathbf{k}\sigma} U_{bg} f a_{\mathbf{k}\sigma}^\dagger a_{\mathbf{k}\sigma} \\ & + 2\nu |\phi_m|^2 - \sum_{\mathbf{k}} \left[(U_{bg} p + g \phi_m) a_{\mathbf{k}\uparrow}^\dagger a_{-\mathbf{k}\downarrow}^\dagger + \text{H.c.} \right] \end{aligned} \quad (15)$$

where $\tau \equiv \pm$ is the index of helicity branch. By defining an order parameter $\Delta = -(U_{bg} p + g \phi_m)$ and rewriting all the field operators in the helicity basis, the total mean-field Hamiltonian becomes

$$\begin{aligned} \mathcal{H} = & -U_{bg} (|p|^2 + f^2) + 2\nu |\phi_m|^2 + \sum_{\mathbf{k}\tau} (\epsilon_{\mathbf{k}\tau} + U_{bg} f) h_{\mathbf{k}\tau}^\dagger h_{\mathbf{k}\tau} \\ & - \frac{\Delta}{2} \sum_{\mathbf{k}} \left[e^{-i\varphi_{\mathbf{k}}} h_{\mathbf{k}+}^\dagger h_{-\mathbf{k}+}^\dagger + e^{i\varphi_{\mathbf{k}}} h_{\mathbf{k}-}^\dagger h_{-\mathbf{k}-}^\dagger + \text{H.c.} \right]. \end{aligned} \quad (16)$$

To determine the variational field parameters (ϕ_m , p , and f), we diagonalize $\mathcal{K} = \mathcal{H} - \mu \mathcal{N}$ by using Bogoliubov transformation and calculate the grand thermodynamic

potential Ω . Here, $\mathcal{N} \equiv \sum_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma}^\dagger a_{\mathbf{k}\sigma} + 2 \sum_{\mathbf{k}} b_{\mathbf{k}}^\dagger b_{\mathbf{k}}$ is the operator of total number of atoms and μ is the chemical potential. Using the field operators for Bogoliubov quasiparticles [33], $\alpha_{\mathbf{k}+}$ and $\alpha_{\mathbf{k}-}$, \mathcal{K} takes the diagonal form,

$$\begin{aligned} \mathcal{K} = & \sum_{\mathbf{k}\tau} E_{\mathbf{k}\tau} \alpha_{\mathbf{k}\tau}^\dagger \alpha_{\mathbf{k}\tau} - U_{bg} (|p|^2 + f^2) + 2(\nu - \mu) |\phi_m|^2 \\ & + \sum_{\mathbf{k}} \left[(\xi_{\mathbf{k}} + U_{bg} f) - \frac{E_{\mathbf{k}+} + E_{\mathbf{k}-}}{2} \right], \end{aligned} \quad (17)$$

where $\xi_{\mathbf{k}} \equiv \epsilon_{\mathbf{k}} - \mu$ and the energies of Bogoliubov quasiparticles $E_{\mathbf{k}\pm}$ are given by,

$$E_{\mathbf{k}\pm} = \sqrt{\left(\xi_{\mathbf{k}} \pm \frac{\hbar^2 \lambda k_{\perp}}{2m} + U_{bg} f \right)^2 + |\Delta|^2}. \quad (18)$$

At temperature T , it is straightforward to write down the grand thermodynamic potential,

$$\begin{aligned} \Omega = & \sum_{\mathbf{k}} \left[\xi_{\mathbf{k}} + U_{bg} f - \frac{E_{\mathbf{k}+} + E_{\mathbf{k}-}}{2} \right] - U_{bg} (|p|^2 + f^2) \\ & + 2(\nu - \mu) |\phi_m|^2 - k_B T \sum_{\mathbf{k}\tau} \ln \left[1 + e^{-\frac{E_{\mathbf{k}\tau}}{k_B T}} \right]. \end{aligned} \quad (19)$$

The field parameters (ϕ_m , p , and f) must satisfy the coupled self-consistent equations, $\partial\Omega/\partial f = 0$, $\partial\Omega/\partial p = 0$, and $\partial\Omega/\partial\phi_m = 0$. Furthermore, the chemical potential is determined by the total number of atoms N , i.e.,

$$N = -\frac{\partial\Omega}{\partial\mu} = 2f + 2\phi_m^2. \quad (20)$$

These four coupled equations can be solved to obtain the pairing order parameter $\Delta = -(U_{bg} p + g \phi_m)$ and chemical potential μ .

IV. RESULTS AND DISCUSSIONS

To clearly contrast the two-channel model with single-channel model, we focus on the resonant limit and neglect the back-ground interaction. By setting $U_{bg} = 0$ (and therefore $\Delta = -g \phi_m$) and renormalizing the energy of molecules ν by using [33],

$$2\nu = 2\nu_0 + \sum_{\mathbf{k}} \frac{g^2}{2\epsilon_{\mathbf{k}}}, \quad (21)$$

we obtain the coupled gap equation and number equation in the two-channel model ($\tau \equiv \pm$),

$$\frac{2(\nu_0 - \mu)}{g^2} = \sum_{\mathbf{k}} \left[\sum_{\tau} \frac{1/2 - n_{\mathbf{k}\tau}}{2E_{\mathbf{k}\tau}} - \frac{1}{2\epsilon_{\mathbf{k}}} \right], \quad (22)$$

$$N - \frac{2\Delta^2}{g^2} = \sum_{\mathbf{k}} \left[1 - \sum_{\tau} \left(\frac{1}{2} - n_{\mathbf{k}\tau} \right) \frac{\epsilon_{\mathbf{k}\tau} - \mu}{E_{\mathbf{k}\tau}} \right] \quad (23)$$

where $n_{\mathbf{k}\pm} \equiv 1/(e^{E_{\mathbf{k}\pm}/k_B T} + 1)$ is the Fermi-Dirac distribution function. In contrast, the gap and number equations in the single-channel model are given by [13],

$$-\frac{m}{4\pi\hbar^2 a_s} = \sum_{\mathbf{k}} \left[\sum_{\tau} \frac{1/2 - n_{\mathbf{k}\tau}}{2E_{\mathbf{k}\tau}} - \frac{1}{2\epsilon_{\mathbf{k}}} \right], \quad (24)$$

$$N = \sum_{\mathbf{k}} \left[1 - \sum_{\tau} \left(\frac{1}{2} - n_{\mathbf{k}\tau} \right) \frac{\epsilon_{\mathbf{k}\tau} - \mu}{E_{\mathbf{k}\tau}} \right] \quad (25)$$

respectively. By recalling from Eq. (9) that the effective s -wave scattering length in the two-channel model is $-4\pi\hbar^2 a_s/m = g^2/(2\nu_0)$, it is clear that the gap and number equations in both models have the same structure. However, additional terms, $-2\mu/g^2$ and $-2\Delta^2/g^2$, appear in the two-channel gap and number equations, respectively. For a finite SO coupling constant λ , if $g \rightarrow \infty$, $2\mu/g^2$ and $2\Delta^2/g^2$ go to zero. Then, the equations of the two models become exactly the same. This is the same as the situation without SO coupling. In other words, the single-channel model and two-channel model coincide with each other in the broad resonant limit, as they should be. However, for a finite channel coupling strength g , if λ is sufficiently large, deep two-body bound state (i.e., rashbon) appears, with a divergent chemical potential ($\mu \rightarrow -\infty$; see Fig. 1(b) below). Thus, we can not neglect $2\mu/g^2$ in Eq. (22) anymore and the two models are no longer equivalent. In this strong SO coupling limit, we anticipate a qualitative difference between the single-channel and two-channel models.

Let us turn to detailed numerical calculations. For simplicity, we consider the case in which the temperature is zero and the system is exactly at Feshbach resonance ($\nu_0 = 0$ and $a_s^{-1} = 0$). To characterize the width of Feshbach resonances, we introduce a *dimensionless* channel coupling constant,

$$g_0 = \frac{2m}{\hbar^2 k_F^{1/2}} g, \quad (26)$$

where $k_F = (3\pi^2 N/V)^{1/3}$ is the Fermi wavelength. We take the energy and length in the units of the Fermi energy $E_F = \hbar^2 k_F^2/2m$ and the inverse Fermi wavelength k_F^{-1} , respectively. The SO coupling strength is measured by the dimensionless parameter λ/k_F . In experiments, the typical magnitude of SO coupling strength is at the order of the Fermi wavelength, i.e., $\lambda = O(k_F)$.

Fig. 1 reports the evolution of the pairing gap (a) and chemical potential (b) with decreasing the resonance width. For comparison, the prediction of single-channel model is also shown by solid lines. As seen from Fig. 1(a), for systems with small SO coupling strength, the single-channel model and two-channel model give the same results when g_0 is large enough (i.e., $g_0 > 20$). In the case of ^6Li atoms with $g_0 \simeq 600$ (the purple dot line in Fig. 1), we can not see the difference from the single-channel prediction. This means that the broad resonance condition is always valid for ^6Li atoms. However, for a smaller

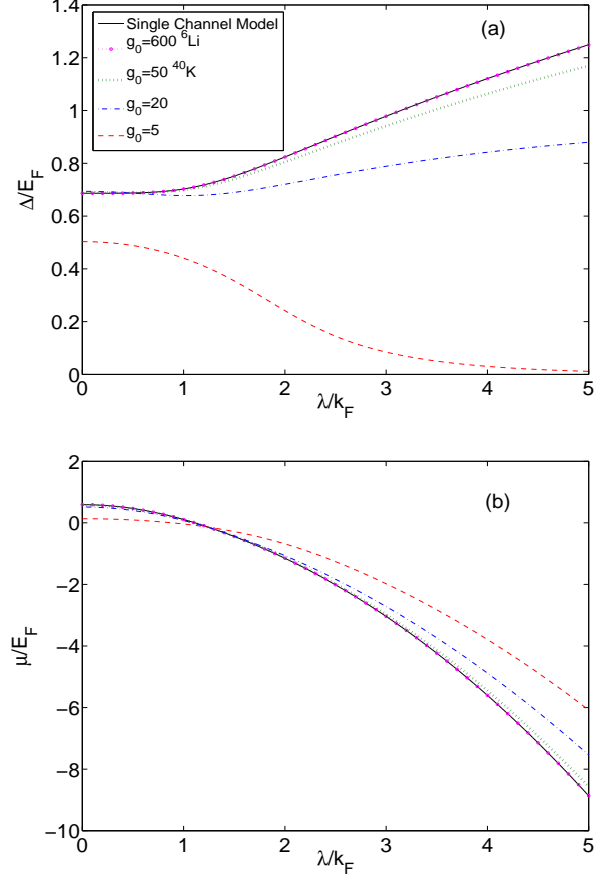


Figure 1: The pairing gap Δ (a) and chemical potential μ (b) as a function of the Rashba SO coupling strength λ for different atom-molecule coupling g_0 or resonance width at zero temperature and at Feshbach resonance.

g_0 , e.g., ^{40}K atoms with $g_0 \simeq 50$, the pairing gap deviates clearly from the single-channel prediction at the typical experimental SO coupling strength $\lambda/k_F = 3$, although the two models give essentially the same pairing gap in the absence of SO coupling. With increasing the SO coupling strength, the difference between the two models becomes more significant. For even smaller g_0 (i.e., $g_0 = 5$), it is interesting that the dependence of the pairing gap on SO coupling strength changes qualitatively. The pairing gap starts to decrease with increasing SO coupling strength and vanishes at sufficiently large SO coupling.

This dramatic change is somehow not anticipated, as the pairing gap is always enhanced by SO coupling in the single-channel model. It is closely related to anisotropic superfluidity caused by the Rashba SO coupling. As discussed in Ref. [13], due to SO coupling the fermionic superfluid has mixed singlet and triplet components. The fraction of triplet pairing grows with increasing the SO coupling strength. Thus, within the single-channel model, the amplitude of pairing gap reflects both singlet and triplet pairing strengths, and increases as the SO coupling increases. In the two-channel model, how-

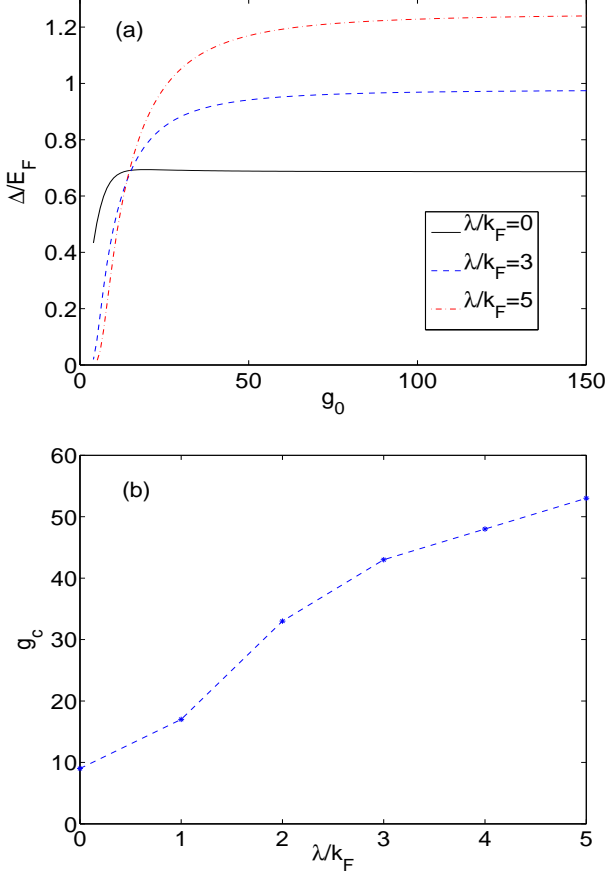


Figure 2: (a) The pairing gap Δ as a function of the resonance width g_0 for different Rashba SO coupling strength λ/k_F . (b) The critical resonance width g_0 (below which the pairing gap differs more than 5% from the prediction of the single-channel model) for different λ/k_F .

ever, the most important resonant-interaction Hamiltonian \mathcal{H}_{am} is of s -wave character and hence favors the singlet pairing. As the triplet pairing is favored by Rashba SO coupling, the resonance width and SO coupling have opposite effects on the pairing gap and destruct with each other. The destruction becomes very pronounced with decreasing the resonance width, leading to a completely suppressed pairing gap at large SO coupling and narrow resonance width.

The suppression of pairing gap can also be mathematically understood from the two-channel gap equation, Eq. (22), where we may treat $2(\nu_0 - \mu)$ as the effective energy detuning of Feshbach molecules. By increasing the SO coupling, the chemical potential will diverge to $-\infty$, and hence the effective energy detuning is pushed up to the BCS limit. As a result, the pairing gap becomes significantly suppressed.

Now, it is clear that the broad resonance limit becomes much more difficult to reach in the presence of Rashba SO coupling. To quantitatively characterize the broad resonance condition, we show in Fig. 2(a) the behavior

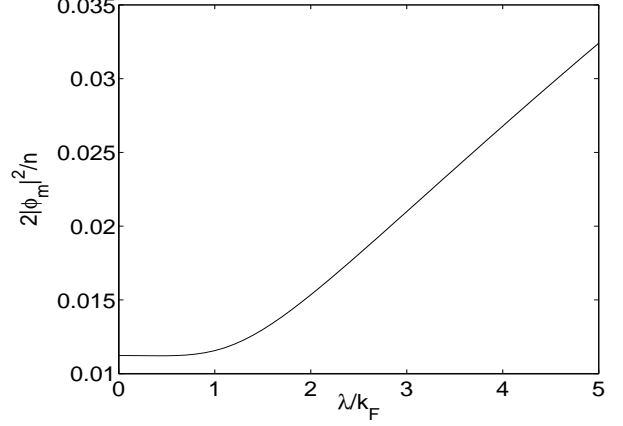


Figure 3: The molecular fraction $2|\phi_m|^2/N$ as a function of the Rashba SO coupling λ/k_F for ^{40}K atoms where $g_0 = 50$.

of the pairing gap Δ as a function of the resonance width g_0 , for some selected values of λ/k_F . As g_0 increases, the pairing gap Δ grows rapidly at first, and then saturates to the prediction of single-channel model. Quantitatively, we may define a critical g_c , in such a way that above g_c the relative difference in the pairing gaps predicted by the two models is less than 5%. Fig. 2(b) presents g_c as a function of SO coupling strength. It gives a qualitative phase diagram. Above g_c we may safely use the single-channel model to describe the Rashba SO coupled atomic Fermi gas near Feshbach resonances. While below g_c , the two-channel model must be adopted. For ^{40}K atoms with $g_0 \simeq 50$, we find that the single-channel model becomes insufficient at a moderate Rashba SO coupling, $\lambda \sim 3k_F$.

V. EXPERIMENTAL RELEVANCE

To experimentally test our predictions, we consider optical molecular spectroscopy, which projects Feshbach molecules onto a vibrational level of an excited molecule. The rate of excitations enables a precise measurement of the fraction of the closed-channel Feshbach molecules in the paired state, although the fraction could be extremely small [36]. Near resonance, the paired state may be treated as dressed molecules [35, 36],

$$|\text{dressed}\rangle = e^{i\phi} \sqrt{1 - Z_m} |\text{open}\rangle + \sqrt{Z_m} |\text{closed}\rangle, \quad (27)$$

where Z_m can be identified as the component fraction of Feshbach molecules, i.e., $Z_m = 2|\phi_m|^2/N$. As an concrete example, in Fig. 3, we show the fraction for ^{40}K atoms (with $g_0 \simeq 50$) as a function of SO coupling strength. As the SO coupling increases, the population of Feshbach molecule is almost flat at first. However, after the coupling reaches a critical value $\lambda \simeq k_F$, it grows very fast.

The impact of SO coupling on the population of Feshbach molecules is best seen in Fig. 4, where we present

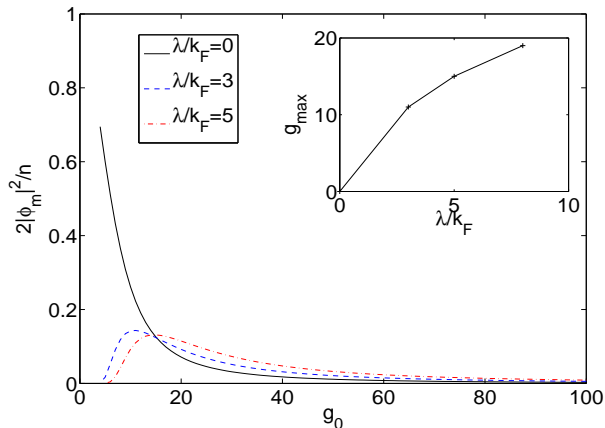


Figure 4: The molecular fraction as a function of the resonance width g_0 for different Rashba SO coupling. The inset shows g_{\max} (see text for definition) as a function of the SO coupling strength.

the fraction as a function of the resonance width at several SO coupling strengths. For a given non-zero SO coupling, the fraction is a non-monotonic function of the resonance width. By decreasing g_0 from the broad resonance limit, the fraction first grows then drops to zero, as a result of the competition between SO coupling and resonance width, as mentioned earlier. In the limit of narrow resonance, the vanishing molecular fraction is consistent with the suppression in the pairing gap at large SO coupling, as shown in Fig. 1(a), due to the relation $\Delta = -g\phi_m$. In contrast, in the absence of SO coupling the molecular fraction increases steadily with decreasing the resonance width. We may define a characteristic g_{\max} at which the fraction reaches its peak value. As shown in the inset of Fig. 4, when λ/k_F is zero, namely there is no SO coupling, $g_{\max} = 0$, and the population reaches unity in the limit of $g_0 = 0$. As the SO coupling increases, g_{\max} increases. We emphasize that for small resonance width g_0 , even a small SO coupling could lead to a strong suppression of the population of Feshbach molecules.

VI. SUMMARY

In conclusion, we have investigated a Rashba spin-orbit coupled Fermi gas near Feshbach resonances, by

using a two-channel model. When the spin-orbit coupling strength is small and Feshbach resonance is broad, the two-channel model is equivalent to the single-channel model, as we may anticipate [34, 35]. However, for a given resonance width, if the SO coupling strength is sufficiently large, these two models are no longer equivalent. Moreover, for a narrow resonance the pairing gap and the fraction of Feshbach molecules are strongly suppressed by SO coupling. We could test these predictions by measuring experimentally the molecular fraction using optical molecular spectroscopy [36]. We have characterized quantitatively the equivalence of the two models by introducing a critical resonance width, above which the two models are approximately the same. By calculating the dependence of the critical resonance width on the spin-orbit coupling strength, we have found that the single channel model may break down for Rashba spin-orbit coupled ^{40}K atoms at a moderate spin-orbit coupling strength.

Our results are obtained within mean-field theory, which is known to provide a qualitative picture of resonantly-interacting atomic Fermi gases. For quantitative purpose, the crucial pairing fluctuation must be included. This may be addressed by using many-body T -matrix theories in the future [35, 39, 40].

Acknowledgments

Jing-Xin Cui and Gui Lu Long were supported by the National Natural Science Foundation of China (Grant No. 11175094), National Basic Research Program of China (NFRP-China Grant No. 2009CB929402 and No. 2011CB9216002), and Tsinghua University Initiative Scientific Research Program. Xia-Ji Liu and Hui Hu were supported by the ARC Discovery Projects (Grant No. DP0984637 and No. DP0984522) and the NFRP-China (Grant No. 2011CB921502).

-
- [1] Y.-J. Lin, K. Jiménez-García, and I. B. Spielman, *Nature* (London) **471**, **83** (2011).
 - [2] R. A. Williams, L. J. LeBlanc, K. Jiménez-García, M. C. Beeler, A. R. Perry, W. D. Phillips, and I. B. Spielman, *Science* **335**, 314 (2012).
 - [3] J.-Y. Zhang, S.-C. Ji, Z. Chen, L. Zhang, Z.-D. Du, B. Yan, G.-S. Pan, B. Zhao, Y. Deng, H. Zhai, S. Chen, and

- J.-W. Pan, *Phys. Rev. Lett.* **109**, 115301 (2012).
- [4] P. Wang, Z.-Q. Yu, Z. Fu, J. Miao, L. Huang, S. Chai, H. Zhai, and J. Zhang, *Phys. Rev. Lett.* **109**, 095301 (2012).
- [5] L. W. Cheuk, A. T. Sommer, Z. Hadzibabic, T. Yefsah, W. S. Bakr, and M. W. Zwierlein, *Phys. Rev. Lett.* **109**, 095302 (2012).
- [6] C. Zhang, S. Tewari, R. M. Lutchyn, and S. Das Sarma,

- Phys. Rev. Lett. **101**, 160401 (2008).
- [7] X.-J. Liu, L. Jiang, H. Pu, and H. Hu, Phys. Rev. A **85**, 021603(R) (2012).
 - [8] X.-J. Liu and H. Hu, Phys. Rev. A **85**, 033622 (2012).
 - [9] M. Z. Hasan and C. L. Kane, Rev. Mod. Phys. **82**, 3045 (2010).
 - [10] X.-L. Qi and S.-C. Zhang, Rev. Mod. Phys. **83**, 1057 (2011).
 - [11] J. P. Vyasankere and V. B. Shenoy, Phys. Rev. B **83**, 094515 (2011).
 - [12] J. P. Vyasankere, S. Zhang, and V. B. Shenoy, Phys. Rev. B **84**, 014512 (2011).
 - [13] H. Hu, L. Jiang, X.-J. Liu, and H. Pu, Phys. Rev. Lett. **107**, 195304 (2011).
 - [14] Z.-Q. Yu and H. Zhai, Phys. Rev. Lett. **107**, 195305 (2011).
 - [15] M. Gong, S. Tewari, and C. Zhang, Phys. Rev. Lett. **107**, 195303 (2011).
 - [16] W. Yi and G.-C. Guo, Phys. Rev. A **84**, 031608(R) (2011).
 - [17] J. Zhou, W. Zhang, and W. Yi, Phys. Rev. A **84**, 063603 (2011).
 - [18] J. D. Sau, R. Sensarma, S. Powell, I. B. Spielman, and S. Das Sarma, Phys. Rev. B **83**, 140510(R) (2011).
 - [19] M. Iskin and A. L. Subaşı, Phys. Rev. Lett. **107**, 050402 (2011).
 - [20] T. Ozawa and G. Baym, Phys. Rev. A **84**, 043622 (2011).
 - [21] K. Zhou and Z. Zhang, Phys. Rev. Lett. **108**, 025301 (2012).
 - [22] L. Han and C. A. R. Sá de Melo, Phys. Rev. A **85**, 011606(R) (2012).
 - [23] K. Seo, L. Han, and C. A. R. Sá de Melo, Phys. Rev. A **85**, 033601 (2012).
 - [24] L. Dell'Anna, G. Mazzarella, and L. Salasnich, Phys. Rev. A **84**, 033633 (2011).
 - [25] L. He and X.-G. Huang, Phys. Rev. Lett. **108**, 145302 (2012).
 - [26] L. He and X.-G. Huang, Phys. Rev. B **86**, 014511 (2012).
 - [27] L. Jiang, X.-J. Liu, H. Hu, and H. Pu, Phys. Rev. A **84**, 063618 (2011).
 - [28] X.-J. Liu, Phys. Rev. A **86**, 033613 (2012).
 - [29] X. Yang and S. Wan, Phys. Rev. A **85**, 023633 (2012).
 - [30] P. Zhang, L. Zhang, and W. Zhang, Phys. Rev. A **86**, 042707 (2012).
 - [31] P. Zhang, L. Zhang, and Y. Dong, Phys. Rev. A **86**, 053608 (2012).
 - [32] S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. **80**, 1215 (2008).
 - [33] M. Holland, S. J. J. M. F. Kokkelmans, M. L. Chiofalo, and R. Walser, Phys. Rev. Lett. **87**, 120406 (2001).
 - [34] R. B. Diener and Tin-Luo Ho, arXiv:cond-mat/0405174v2.
 - [35] X.-J. Liu and H. Hu, Phys. Rev. A **72**, 063613 (2005).
 - [36] G. B. Partridge, K. E. Strecker, R. I. Kamar, M. W. Jack, and R. G. Hulet, Phys. Rev. Lett. **95**, 020404 (2005).
 - [37] Z. Zheng, M. Gong, X. Zou, C. Zhang, and G.-C. Guo, arXiv:1208.2029 (2012).
 - [38] L. Dong, L. Jiang, H. Hu, and H. Pu, arXiv:1211.1700 (2012).
 - [39] H. Hu, X.-J. Liu, and P. D. Drummond, Europhys. Lett. **74**, 574 (2006).
 - [40] X.-J. Liu and H. Hu, Europhys. Lett. **75**, 364 (2006).